Enlarging Mathematical Activity From Modeling Phenomena to Generating Phenomena[1]

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Traditionally, mathematics has been used as means for modeling aspects of the experienced world, and it is often taken as axiomatic that one can learn mathematics more effectively if one is able to apply what one already knows and can do. We illustrate how we can substantially deepen the connection with everyday experience by using mathematical functions to generate phenomena as well as model them. We first provide a framework for examining relations among simulations, notations and physical phenomena. We then illustrate with a 9th grade classroom episode how students' activity taps into their linguistic, kinesthetic and notational resources to deepen their engagement with important mathematical ideas.

Introduction

This paper builds upon prior work (Kaput & Roschelle, 1997; Nemirovsky, Tierney, Wright, in press) to which the reader is referred for background and particular features of the learning environments that we are using. For the reader's convenience we quote an edited abstract from Kaput & Roschelle (1997):

> We address the question of how we might exploit interactive technologies to democratize access to ideas that have historically required extensive algebraic prerequisites. Illustrations will be drawn from work in the authors' ongoing SimCalc Project, which builds and tests software simulations, physical devices, and related visualization tools intended to render more learnable the ideas underlying calculus and the Mathematics of Change & Variation beginning in the early grades. We will reflect on how such technologies can change the experienced nature of the subject matter by tapping more deeply into students' cognitive, linguistic and kinesthetic resources. Substantial reorganizations are possible of curricula that have been taken as given for centuries (p.105).

Our purpose here is to focus on an important new affordance of technological learning environments, the ability to *generate* and not merely to *model* phenomena. These environments enable the user to embody their intentions and conjectures within phenomena that he/she can control and interact with in new ways. For example, a student can use mathematical notations or other controls within a computer

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environment to control the motion of physical "Minicars" that move on tracks. The same means can also drive a simulation "within" the computer. Our goal is to understand how to use the rich learning opportunities that emerge as we combine these 2 kinds of phenomena-generating capabilities with each other and with traditional affordances such as linked representations & MBL devices.

We will first briefly describe the theoretical and technological contexts for our work, and then sketch a classroom episode that illustrates the fundamental issues that emerge in such learning environments. An enlarged version of this paper will be available at the PME-22 Meeting, as will videos of the episode and demonstrations of the devices described.

Activities Crossing Notational, Simulation & Physical Realms

Historically, we have always assumed that mathematics was to be used to model and make sense of situations and phenomena - in Yerushalmy's (1997) words "thinking about one thing in terms of simpler, artificial things" (p.165). The thing to be modeled is taken as having an existence independent of the model. But, as we turn to computer-based learning environments, we need further distinctions.

1. Phenomena "inside the computer" vs physical phenomena "outside the computer."
2. "Target phenomena" taken as the subject of mathematical description or control (e.g. motion) vs "notational phenomena" embodied within notations that may be used to describe or control the former.

Clearly, distinctions between targets and notations are relative to intentions at hand. We are using them for heuristic purposes to help expose structures of activities that involve coordination across the different realms of phenomena reflected in Fig. 1, whose arrows refer to electronically realizable connections. This diagram points to an extremely rich set of activity-possibilities while simultaneously omitting some of the most important features of all of them, such as: 1) human interaction: talk, gesture, reflection, conjecture, imaging, comparing, explaining, etc. - the medium in which the activities pointed to come alive. And 2) a related set of "invisibles" - the resources that students bring to the activities, the at-hand results of their everyday experience with language, symbols, space, time, objects, motion, and so on.

We now examine the structures of activities that the diagram points to, and in so doing contextualize some traditional work involving computer technology and reveal some special features of our own work.

Expanding Realms of Activity-Structures

Expanding linked representations in the notational realm to Rate-Totals links

Many researchers, too numerous to mention, have given attention over the past 10 years to the use of multiple linked representations of functions. In terms of Fig. 1, this work concerns the inside of the upper left circle, although in practice much of the work also involved modeling of situations, usually given independently from the computer environments in text form, an exception being Yerushalmy (1997), some of whose software incorporated text fragments as a fourth linked notation system.

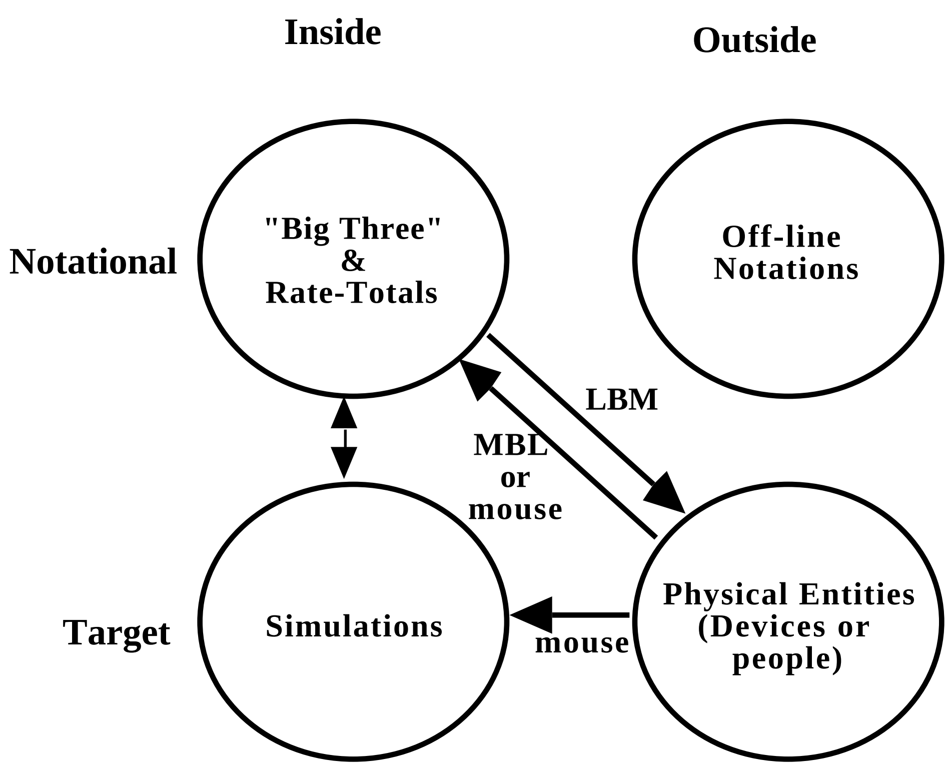

Figure 1

Within the linked representation view, we take the perspective that *understanding the relations between rates and totals descriptions of varying quantities* (and the situations or phenomena that they describe), is a fundamental aspect of quantitative reasoning as it relates to the MCV. For example, to understand the difference between linear and quadratic growth as traditionally approached, one looks at the shapes of the respective graphs, or the growth patterns in their respective tables, in relation to the corresponding equation. But from our perspective, the linear and quadratic functions must also be seen as related by the rates-totals connection. The quadratic is the accumulation of linearly changing quantities, and the linear is the rate of change of quadratically changing quantities. We believe that these kinds of connections allow a much deeper understanding of the basic mathematics than do approaches that ignore them. We not only can connect graphs and formulas, we can cross-connect, for example, a rate graph to a totals formula. Physical realizations of rate-totals connections can take many forms, but we take the historical starting point, the velocity-position connection in motion phenomena.

Expanding the realm of simulations to MBL Data

Simulations involve the top-down arrow in Fig. 1. Such a graphically-oriented software context, called MathWorlds (http://www.simcalc.umassd.edu), was discussed in Kaput & Roschelle (1997). A somewhat more formula-oriented environment, is called AlgebraAnimator (Logal, 1995). And over the past two decades Microcomputer Based Laboratory (MBL) devices (Tinker & Thornton, 1994), have become more common, and are represented by the arrow from the lower right to the upper left in Fig. 1. We and our colleagues have made extensions of this functionality to enrich the work with simulations (see Fig. 5 in Kaput & Roschelle, 1997) to enable the student both to increase the intensity and personal intimacy with the mathematical representations (Middleton & Kaput, in preparation) as well as to create new situations such as parades or dances that embed function-descriptions of motion in interestingly complex relations. In Fig. 1 these constitute an extension bridging the lower right circle with the two on its left.

Expanding the realm of physical action and connections among notational, cybernetic and physical phenomena

Nemirovsky (1993) has explored numerous ways to expand the realm of physical action by designing devices that allow students to control air flow, the shape of a surface, or rotary motion - either directly by acting on the physical device, *or by using a mathematical function defined on a computer that "drives" the device.* This functionality, now common in industrial manufacturing situations, is represented by the upper to lower-right diagonal arrow labeled "LBM." ("Lines Become Motion.") These devices have been used for investigating how students, explore, think and learn about ideas of rate and accumulation (Nemirovsky and Noble, 1997).

This phenomena-generating capability turns a fundamental relationship between mathematics and experience from one-way to bi-directional, which in turn supports a much tighter and more rapid interaction on which to base learning. Because the mathematical notation that controls a phenomenon also models it, one can test a model immediately, as we see below. Critically, the student's intentions can be made visible, explicit and testable through the phenomena that the student controls.

Our major goal is to understand how the two forms of phenomena-generation activity - simulations and physical devices - can best be used in combination and in different kinds of classroom situations, optimally exploiting the strengths of one to compensate for weaknesses in the other, as well as build on mutual strengths.

Case Study Illustrating Interactions Among Physical, Cybernetic and Notational Spheres of Activity

The episode took place in the 9th grade algebra class of Michigan, USA teacher, Kellie Bachman, who was participating in a series of experimental activities directed by her colleague Marty Schnepp. In front of the classroom was a car on an inclined plane (Fig. 2 A - upper) connected via MBL to a computer that displays its velocity in real time (Fig. 2 B), and an LBM "Minicar" (Fig. 2 A - lower) whose motion can be controlled by a graph drawn on the same computer screen.

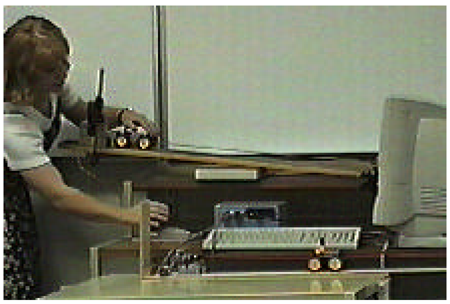

Figure 2 A

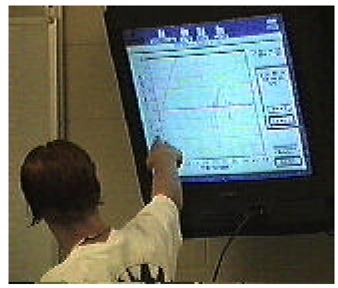

Figure 2 B

The Episode

We will briefly describe the first 30 minutes of the session. The teacher began the class by reviewing a homework task, which consisted in creating qualitative graphs describing the speed of a sled going up and down a hill. She drew 3 different graphs that students had proposed:

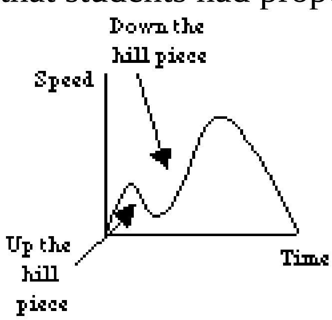

Figure 3

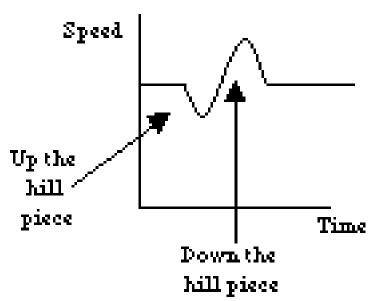

Figure 4

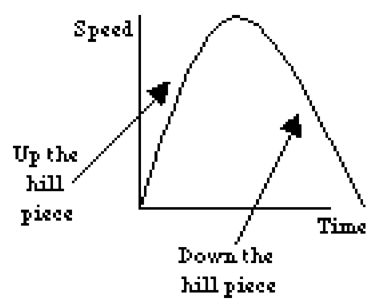

Figure 5

The teacher proposed to measure the speed of the car on the inclined plane to generate a graph that would correspond to the "downhill" part. The graph that appeared on the computer screen as the car rolled down was similar to Fig. 6 (the car bounced back a few times as it reached the end of the inclined plane). The immediate discussion focused on the negative velocity peak. Students commented: "it hit a tree or something," "it bounced back." Students proposed some experiments to verify that the meaning of negative velocity was "going backwards." The first one was to move the car by hand to see whether the velocity becomes negative. Then they wanted to see if they could "catch" the car by hand on its way down, preventing it from bouncing upwards. This proved to be difficult because one tends to stop the car rolling down by slightly pushing it upwards; this subtle effect that became apparent in the resulting velocity graphs with short negative sections.

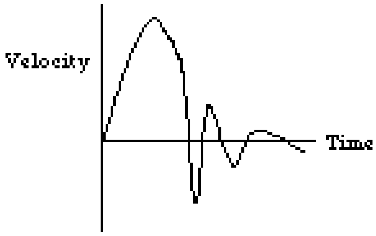

Figure 6

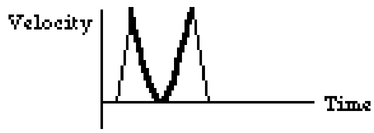

Figure 7

Right after one student succeeded in stopping the car without producing negative velocity, the class discussed small "bumps" that appeared on his velocity graph after the car had been "stopped." Another student explained the bumps: "it is because he stopped it, let it move [the car], stopped it, let it move, until it [the car] got to the bottom [of the inclined plane]." Students accompanied these descriptions by moving their hands in the manner they thought had generated the small bumps. To see more clearly the effect of these small bumps, the teacher proposed to move the Minicar driven by the same velocity graph that included the "little bumps," transforming the velocity *model* to a velocity *generator*. Because the motor makes a sound whose pitch is proportional to its speed, the teacher advised to "listen carefully to the sound of the motor." (Variations of speed that are difficult to see are often easy to hear). The Minicar reproduced in its motion the "bumpy" slow motion that the student had produced by hand to let the car reach the bottom of the inclined plane. Inspired by this experiment, students now wanted to make the Minicar reproduce the original motion of the car rolling down on the inclined plane when it "bounced back."

This was an opportunity for the teacher to discuss again the interpretation of Fig. 6. Since the students kept referring to this graph as the one where the sled had "hit a tree," she asked exactly at which point on the graph the sled had hit the tree. Some thought that it was at the low extreme of the negative peak, others when the graph crossed the horizontal axis. After running the Minicar and reaching an apparent consensus on the latter, the teacher asked whether they wanted to make other experiments. One student wondered how the Minicar would "do" if one drew a velocity graph for it that did not start at zero velocity. How would the Minicar realize such a request? They tried it and saw that even before running the car, the software added an initial piece to the drawn graph so that it would start from zero. The student who had proposed Fig. 4 for the homework said that a velocity graph could, however, start from a non-zero value; one could, say, start the graph when one is walking toward the hill with the sled. On the other hand "if you start from sitting at the bottom of the hill I agree" that the graph had to start at zero velocity.

The teacher then proposed to discuss Fig. 5. Several students said that the graph should be inverted (uphill slowing down and downhill speeding up). When the inverted graph was drawn on the computer to drive the Minicar, the software modified the "U" shape producing Fig. 7. At this point the class had no difficulty in interpreting why the software had "added" the initial and the final segment. After observing the Minicar enacting Fig. 7, two students made the following comments:

Jesse: When the graph is going up, the sled is going down. When the graph is going down, the sled is going up or something. That's confusing, because...

Ben: Just think about it as the speed going up, not the sled going up. We are talking about the speed and the time, not the sled. It has to do with the hill because it has to do with what part of the hill you are on, but it is really about the speed.

Analysis

Three aspects of this episode seem of particular relevance for our research question: *How can technologies and learning environments change the experienced nature of the MCV by tapping more deeply into students' cognitive, linguistic and kinesthetic resources?* These 3 aspects suggest some of the ways in which the interplay among notations, simulations, and physical phenomena can be productively expanded: by incorporating the kinesthetic activity, empowering notations to create phenomena, and testing similarities and differences between virtual and physical phenomena.

1. *Incorporating the kinesthetic activity.* The car rolling downward displayed its corresponding graph showing, for the first time in this class, the possibility of negative velocity. To account for this new possibility the students did a number of experiments moving the car by hand. The task of stopping the car without generating a negative velocity became an engaging *kinesthetic* challenge. Note how negative velocity developed a significance that went beyond the mere statement of "it means going backwards." It brought to the fore the common but ignored fact that the act of "stopping" a motion ordinarily generates a damped oscillation.

2. *Empowering notations to create phenomena.* Controlling the Minicar with the "bumpy" graph generated by hand not only confirmed that the velocity bumps represented slow and discrete motions, but gave to the graph, as a symbolic notation, a different status: it can create phenomena. The graph not only represented how the student had moved his hand, but it was also "empowered" to drive a physical car. An indication of this new relevance is the spontaneous student requests to run the Minicar according to other graphs that had been discussed before.

3. *Experimenting with similarities and differences between virtual and physical phenomena.* Even though this particular class was not using simulation software, there were pervasive "imaginary" simulations in their talk about the sled going up and down

the hill, hitting a tree," sitting at the bottom of the hill," etc. The discussion of starting at non-zero velocity made prominent some of the differences between cybernetic and physical phenomena. While the students could imagine a velocity graph starting at non-zero velocity by beginning the graph as one "is already walking," the graph driving the Minicar – subject to the constraint that it must describe the *entire* motion of the car - is *forced* to start at zero velocity.

Overall, the students and teacher seemed fluently and easily to combine the three realms in Fig. 1 (the 4th, Offline Notations, appeared in the homework). They did not seem confused or troubled by referring to the car on the inclined plane as hitting a tree that was not there or by using the same graph to symbolize different actions. This seems to indicate that these tools and learning environments help to recruit linguistic and cognitive resources from their everyday experience.

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